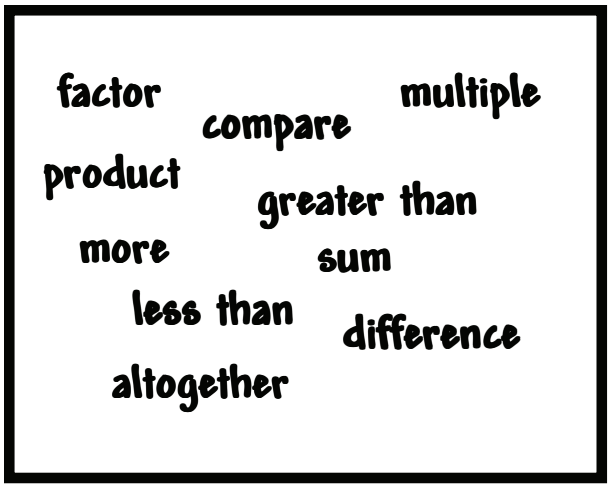
Using Student-Created Problems as a Tool for Teaching and Evaluating Problem Solving

Students who are able to create and solve their own problems often demonstrate a far deeper understanding of mathematical concepts than do those students who are only able to solve teacher-generated problems. Whenever you ask students to create their own problems, it is **important that you establish criteria that require students to show solutions to their own problems**. Students often find problems generated by their peers more engaging than teacher-generated problems. Peer-generated problems reflect student vocabulary and interests, while also often employing familiar student names or amusing, situational storylines, thus creating meaning in the learning process.

Step One

Create a word wall listing mathematical words and terminology. Add new terms as they are introduced in class. Use the words generated on the word wall in your language and spelling programs wherever appropriate.



Step Two

Whenever a student employs a word shown on the word wall in a problem or a number sentence, award that student one bonus mark.

Step Three

Introduce criteria that describe the writing of effective problems:

- ✓ The problem is somewhat challenging (it is not too easy)
- ✓ The problem is easy to read and makes sense
- ✓ All of the clues included are necessary and are not superfluous
- ✓ Mathematical terms are used properly

Example: **Making a Problem Using the Multiplication Facts**

The example that follows assumes that students have already been taught the **multiplication operation** and some of the multiplication facts. Begin by teaching the two-times table and reviewing the related terms. If students have not been taught the multiplication facts, review the methods outlined in the ***Numeracy*** section of this manual or refer to the ***Power of Ten Learning to Multiply and Divide Using Visual Tools and Patterns***.

- **3 x 2 =** ask for the answer
- **4 x 2 =** ask for the answer
- **5 x 2 =** ask for the answer but now begin to refer to the answer as the "product"
- Ask for the next product **(12)**
- Ask for the next product **(14)**
- Then ask: What is the first factor? The second factor? *(Note: Underline the factors as you write them on the blackboard.)*
- Repeat the process for **16** and **20**.

When sufficient examples are listed on the blackboard, and at least half of the class is able to see the pattern emerging in the table, **ask if any student is able to state a rule or theory that describes the pattern** shown in the table. Where a student provides a viable rule, award that student an oral (unrecorded) mark of ten. Wherever a student employs a word from the word wall, award one additional oral mark for each word selected. Wherever a student introduces a new mathematical word for the word wall, award that student two bonus marks. Where students maintain daily journals, ask any student who provides a viable rule or theory to write his rule in his personal journal.

For example: where a student may state the rule for the example above as: *"The answer ends in two, four, six, eight or ten"*, award an oral mark of ten. Ask if any student is able to "say the rule" in a different way. Where another student may then say: *"The product ends in an even number"*, award that student thirteen marks (ten marks for recognition of the stated rule, one bonus mark for the term "product", and two bonus marks for introducing the term "even".) Then add the word "even" to the word wall. Each student who has participated in the brainstorming process and has offered a stated rule to class discussion must then record his rule and related mark in his personal journal. **Ensure that you review all student journals prior to report card writing.** Where you are using an anecdotal reporting process (as a support to a letter-grade or check-list reporting process) the specific rules generated by students may be cited as evidence of that learner's developing mathematical thinking and understanding.

Try to elicit at least one rule or pattern from each student each school term. You may choose to record all rules or terminology generated by individual students on the ***"Class Record For Problem Solving and Pattern Recognition"*** sheet found on *page 61*.

Repeat the process outlined above with the five, zero, one, and ten-times tables. Then ask students to create one or two problems that relate to the multiplication operation. Provide the following example of a possible "frame" for writing such problems:

I am thinking of a question.
It is a multiple of 4.
The answer is greater than 30 and less than 40.
The sum of the digits in the product is 5.
What is my question?
(The equation is 8 x 4 = 32)

The following problem was created by *Alison James*, a grade-three student:

I am thinking of a multiplication question.
The answer is greater than 20 and less than 40.
The sum of the factors is 11.
The difference between the factors is 1.
What is my question?
(The equation is 6 x 5 = 30)

Scoring Student-Generated Problems

Both of the problems listed on the previous page adhere to all suggested criteria and therefore justify a total full score of five marks out of five. Alison would have scored six marks had she used the term "product" instead of the word "answer". As Alison's question contains one unnecessary clue, an uncompromising evaluator may have decided to deduct half a mark from her final score for having included extraneous information. However, the real world often produces problems with too much information. Always remain reasonable and supportive. **Focus on strengths and acknowledge weaknesses.**

Establishing Criteria for Student Answers

Establish the **two criteria listed below** when assessing student answers.

I am thinking of a question.
It is a multiple of 4.
The answer is greater than 30 and less than 40.
The sum of the digits in the product is 5.
What is my question?
(The equation is 8 x 4 = 32)

Criteria 1:

Evidence of student thinking must be clearly demonstrated. Other individuals should be able to follow the work provided. The work below clearly demonstrates mathematical reasoning generated in response to the problem listed above:

4, **8**, **12**, **16**, **20**, **24**, **28**, **32**, **36**, **40**
32, **36**
3 + 2 = 5

Criteria 2:

The answer must be stated in a sentence.

The answer is 8 x 4 = 32.

Having established the criteria outlined above, follow **by asking students to write their own questions**. When you have a substantial supply of student-generated questions, distribute questions by either writing them on the board or printing them on activity sheets. Student-generated questions may also be used effectively in tests. Always ensure that you acknowledge the writer of each question you use, as in this way you will provide valuable positive feedback and reinforcement to each student thus publicly recognized.

Alternate Criteria for Scoring Solutions Found

Criteria 1:

Evidence of student thinking must be clearly demonstrated. Evidence must address the following question: ***How did the student arrive at the answer?***

- Award **five out of ten possible marks** for showing any evidence that may lead to a solution.
- Award **up to three marks** for showing other steps.
- Award **two marks** for writing the answer in a clear statement.

The advantage of the "five out of ten" initial score (awarded for any beginning attempt) is that it rewards "getting started", which is an initial stumbling block for so many students. Many students hesitate to commit any initial thought or mathematical operation to paper, fearing public or peer ridicule. Many others remain convinced that their attempts will inevitably fail to yield a successful solution, and they therefore remain trapped by a sense of learned helplessness and lack of personal power.

Criteria 2:

Some assessment processes employ a zero, one, or two-mark criteria system.

- **Zero is awarded** where no work is shown that may lead to a solution.
- **One mark is awarded** where some evidence of work that may lead to a solution is shown
- **Two marks are awarded** for a correct solution, or for an almost correct solution, where an obvious "slip" or inconsequential error shows.

Facilitating Student-Generated Problems

Supply students with the "frame" that follows on page 67 when they write their own questions. Photocopy the following page and then cut the page in half. Instruct students to write a question on one side of the sheet and then to show the steps necessary to reach a solution on the reverse of the sheet. **Ensure that students show their work.** When students are fully conversant with all assessment criteria, instruct students to mark their own work and to evaluate their own progress. The "frame" sheet that follows may be used most effectively in conjunction with the ***Weekly Graph***, ***Sports and Olympics***, ***The Nutrition Project*** and the ***Height Project***.

Struggling Students:

Some students have difficulty writing clues for a question. Have them start by writing the answer and then generating a problem.

Example:

Answer: 4 x 3 = 12

Clues:

1. Product is greater than **10** and less than **16**.
2. Product is even.
3. Product is a multiple of **3**.

What is my question?

I am thinking of a question. Use my clues to figure it out.

Did you use any of these words: factor, multiple, product, sum, odd, even, difference, more, less, compare, greater than, less than?

Write your answer and show how you arrived at your answer on the back of this activity sheet. Can't think of any clues. Write the **answer** on the back, then write the clues.

Self Evaluation:	
Written Problem	**Answer**
0 1 2 3 4	**0 1 2 3 4**

Teacher Evaluation:	
Written Problem	**Answer**
0 1 2 3 4	**0 1 2 3 4**

What is my question?

I am thinking of a question. Use my clues to figure it out.

Did you use any of these words: factor, multiple, product, sum, odd, even, difference, more, less, compare, greater than, less than?

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Self Evaluation:	
Written Problem	**Answer**
0 1 2 3 4	**0 1 2 3 4**

Teacher Evaluation:	
Written Problem	**Answer**
0 1 2 3 4	**0 1 2 3 4**